

Uncertainties in LCA (Subject Editor: Andreas Ciroth)

Possibility Theory: A New Approach to Uncertainty Analysis?

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Abstract

Background, Aims and Scope. The problem of the evaluation of practitioner's belief and belief-related uncertainties on LCA results obtained from different methodological choices has been addressed so far by scenario modeling, Cultural Theory perspectives and probabilistic simulation. The direct evaluation of belief and related uncertainties could be of interest, e.g. when the information available (resulting from classical uncertainty analysis or the application of the precautionary principle) do not allow one to choose between methodological alternatives leading to different LCA results and conclusions. The difficulty of modeling belief arises from the additive nature of classical measures, e.g. probabilities. Since the 1960s, non-additive measures (e.g. possibilities) have been developed and applied to model belief in real world problems. The aim of this paper is to discuss the application of possibility measures in LCA for uncertainty analysis in complement to classical approaches.

Methods. The nature and the meaning of possibilities are briefly introduced by comparison with probabilities (subjective or not) in order to enlighten strengths, drawbacks and complementarities. A tentative possibilistic approach based on the evaluation of a posteriori possibilities of final LCA results depending on a priori possibilities of the methodological choices behind the calculations is described, also by means of an application example.

Results and Outlook. A new approach for the modeling of practitioner's belief and belief-related uncertainties in complement of classical methods of uncertainty analysis has been proposed for discussion. Uncertainty can be characterized by confidence intervals and indexes that could help practitioners in making methodological choices and could improve the interpretation and reliability of LCA results, still increasing its sophistication.

Keywords: Belief; fuzzy sets; methodological choices; possibility theory; probability theory; uncertainty analysis

Introduction

The evaluation of practitioner's belief and belief-related uncertainties of LCA calculations and results could be of interest in the LCA practice, e.g. when the information available do not allow one to choose between methodological alternatives leading to different conclusions, and could inform about their reliability. So far, the evaluation of belief-related uncertainties resulting from methodological choices, both

in Life Cycle Inventory (LCI) and in Life Cycle Impact Assessment (LCIA), could be addressed by identifying the relevant alternatives and performing sensitivity analysis by scenario modeling (Huijbregts 1998, Björklund 2002), by using Cultural Theory perspectives (Hofstetter 1998) and by probabilistic simulation (Goedkoop et al. 2000). Scenario modeling show how results vary depending on the methodological choices, but do not allow evaluating belief-related uncertainties of each. Cultural Theory perspectives are fixed and it is sometimes difficult to relate them to the practitioner's belief in specific choices. Probabilistic simulation is a detailed approach and it is considered a possible way to evaluate belief-related uncertainties affecting LCA results. These approaches could add significant sophistication to calculations and they are not such a common practice (Ross 2002). So, at a first glance, by accepting increased sophistication, it seems that the lack or different types of information, subjectivity of beliefs and uncertainty related issues could all be formulated in terms of probabilities in view of their aggregation and/or Monte Carlo modeling. This approach is already well known and also complicated enough. Thus, a possible question is: is there the need for invoking additional sophistication by the use of obscure Possibility Theory principles? Actually, in our opinion, the need for invoking a complementary (not concurrent) approach could exist due to the axiomatic limitations of probabilities in belief-related calculations and uncertainties.

1 Possibility vs. Probability?

Historically, the difficulty of modeling belief is mainly due to the lack of non-additive measures. Subjective probabilities, i.e. a priori probabilities chosen on the basis of the belief and expertise of practitioners (and thus referring to the Bayesian framework), could be used but they still remain additive measures. This means that for independent events, the belief in the union of two events is the sum of the beliefs in each of them. For dependent events, the calculation of the belief in the union or intersection of the two is more complicated and conditional probabilities have to be evaluated. Events are not always independent in real-world problems and also in LCA, where information can be imprecise but coherent. In addition to the difficulty to prove the independency between events, it is even more difficult to identify all the possible events, as the probabilistic framework requires.

Consider, for example, several LCI emission contributing to the Greenhouse Effect. The uncertainty affecting the LCI results concerns their value and their trueness. The uncertainty on the values depends on the measurements and procedures by which they have been calculated and on natural variability. Measurements and procedures could be considered as independent from the subjectivity of practitioners and from natural variability, then the independency of the events to be aggregated can be proven. The uncertainty can be characterized by assigning a (frequentist) probability distribution to each LCI emission and then propagated to evaluate the uncertainty on the LCIA result by MonteCarlo Analysis or by interval calculation. Probability theory is well suited to represent precise results of mutually exclusive (independent) events, that is the case of an emission value that can not be (in physical terms), e.g. 30 kg and at the same time 31 kg. From measurement campaigns, probability distributions state how likely the emission could be 30 or 31. The likelihood of the LCIA result is calculated accordingly. The uncertainty on the trueness of LCI emissions (how likely they are, according to the practitioner's belief) depend on the belief in the methodological choices behind their calculation, e.g. cut-off and allocation rules, data estimation and product system modeling. The independency of such events is more difficult to be proven. Since it is unlikely to assign a probability distribution (or value) directly to each LCI result (it would sound rather arbitrary), a priori probability distributions could be assigned to the methodological choices behind, by means of calculation rules to be defined, and then aggregated. Thus, an LCI emission could be, at the same time, both 30 kg and 31 kg, e.g. depending on cut-off rules and system modeling. Given the dependency between the choices, it is difficult to evaluate the probability of the union and intersection of events. Also, the belief in the LCI results should then be aggregated with the belief in LCIA calculations (choice of the characterization factor, ...) and, again, the choices are not independent. The conclusion is that the modeling of belief by means of probability distributions should be carefully evaluated on a case per case basis.

To overcome these limitations, other measures, called 'belief measures' or 'fuzzy measures', have been developed since the 1960s, built on a weakness of the axiomatic of probability and leading to a family of non-additive measures (Klir 1994). Among them, possibility measures led to successful and numerous industrial applications, e.g. concerning electrical appliances, medical image, control and command of a car. Given the non-additivity of measures on a union of non-intersected events, a possibility distribution $y = r(x)$ on $\langle X, Y \rangle$ (usually $Y \in [0;1]$) looks like a probability one, but any addition in probability theory can be replaced, in possibility theory, by a maximum and any product by a minimum (Dubois et al. 1988). Also, two dual measures (instead of one) are considered to represent the belief: possibility (Pos) and necessity (Nec). Pos equals the maximum of the possibility distribution, i.e. $\text{Pos} = \max[r(x)]$ and $\text{Nec} = 1 - \max[1 - r(x)]$. The rules of Pos calculation state that, in order to obtain the union of two events, it is enough to obtain the simpler between the two, that is coherent with the meaning of 'possible'. Also, the axiomatic defines that, if an event is certain,

then it is also necessarily true and an event is necessary when the opposite is impossible. Two measures (Pos and Nec), instead of only one (probability), are supposed to be more informative about the belief, for the interpretation of results and for decision-making. Nevertheless, the elicitation of possibility values from practitioner knowledge could be difficult due to the novelty of the axiomatic. To simplify the elicitation, subjective probability density functions can be considered to represent a practitioner's belief in methodological choices and then transformed in possibility distributions for calculation (e.g. Geer et al. 1992). Actually, representing beliefs by means of subjective probabilities is fully pertinent, only their aggregation and propagation present the aforementioned limitations.

The conclusion is that possibility theory seems to offer pertinent principles to model a practitioner's belief in LCA results, i.e. uncertainties due to methodological choices. Probability theory can be better suited to model uncertainties on the values of LCA results. The two formalisms complement each other and could be considered simultaneously.

2 A Tentative Possibilistic Approach to Uncertainty Analysis (with example)

Consider the assessment of the Greenhouse Effect due to a CH_4 LCI emission. Two values are calculated according to different methodological choices, 1.5×10^4 kg and 2.50×10^4 kg, leading to two LCIA results, 34.5×10^4 kg CO_2 eq. and 57.5×10^4 kg CO_2 eq. Suppose that the choice between the two LCI values could significantly affect the final conclusion of the LCA study. Apart from the precautionary principle (sometimes misleading), suppose that the uncertainties on LCI values are similar so that the belief in the LCI results (reflecting practitioner's knowledge) could help to make the choice according to the information available. Three events are considered: the choice of the LCI result (X_1 and X_{1b} , respectively for 1.5×10^4 kg and 2.50×10^4 kg), the choice of the characterization factor's (GWP) value (X_2) and the choice of the (linear) model for impact assessment (X_3 and X_{3b} , respectively for 34.5×10^4 kg CO_2 eq. and 57.5×10^4 kg CO_2 eq.). A tentative possibilistic approach to model the belief in LCIA results could be the following:

1. to assign possibility distributions to X_1 , X_{1b} , X_2 , X_3 and X_{3b} representing the a priori practitioner's confidence on the trueness of the events (i.e. not related to the uncertainty of their values). X_1 and X_{1b} are depicted in Fig. 1. The practitioner seems to be more confident that the emission could be 1.5×10^4 kg, but the good confidence on X_{1b} does not prevent considering 2.50×10^4 kg for calculation. The shapes of X_2 , X_3 and X_{3b} are obtained from a priori probability distributions from literature (Huijbregts 1998, Hofstetter 1998). The distribution of X_2 (mean = 23; standard deviation = 0.5) is the same for the two LCI results, because the belief in the GWP should not change. The distributions of X_3 and X_{3b} are centered on different abscissas (34.5×10^4 kg CO_2 eq. and 57.5×10^4 kg CO_2 eq., respectively) but have the same shape (mean = 34.5×10^4 kg and 57.5×10^4 kg; standard deviation = 5000) because the linear model for calculation is the same.

2. to choose a value x_1 to be considered for the propagation of belief; $x_1 = 1.5 \times 10^4$ kg leading to $x_3 = 34.5 \times 10^4$ kg CO₂ eq. and $x_{1b} = 2.50 \times 10^4$ kg leading to $x_{3b} = 57.5 \times 10^4$ kg CO₂, are chosen, since it would be the case if no belief analysis was performed.
3. to use an operator to obtain the a posteriori possibility distribution X_2^* , i.e. the belief in the results of the first calculation considering the a priori beliefs in X_1 and X_2 (Gupta 1991); in this paper, the 'force implication' operator is used (Dujet et al. 1995): $r_{X_2^*}(y) = r_{X_1}(x_1) \times [1 - \text{abs}(r_{X_1}(x_1) - r_{X_2}(y))]$.
4. to consider the abscissa of the centroid of X_2^* and then to start again from point 2 for the calculation involving X_2 and X_3 .

Performing steps 1 to 4 for the two LCI results lead to two distributions, X_3^* and X_{3b}^* . Each abscissa of the distributions is a possible LCIA result given x_1 (x_{1b}) and the corresponding y-coordinate represents the practitioner's confidence on it. Two confidence intervals [Nec, Pos] are obtained: [0.11, 0.76] for X_3^* and [0.05, 0.89] for X_{3b}^* . The lower limits state to what extent the LCIA results are forced to occur. The upper limits state to what extent the results are possible. Closer are the Nec and Pos measures, closer to the probability one is their meaning. In order to consider at the same time the information given by Pos and by Nec measures and to simplify the interpretation, a confidence index $C = \text{Pos} + \text{Nec} - 1$ is calculated, where $-1 \leq C < 0$ represents the degree of confirmation of the LCIA result by the evidence and $0 < C \leq 1$ represents the degree of disconfirmation of the LCIA result by the evidence (Geer et al. 1992). The higher the absolute value of C , the higher is the degree. In this example, $C = -0.13$ for X_3^* and $C = -0.06$ for X_{3b}^* . The conclusion is that, according to the beliefs in X_1 , X_{1b} , X_2 , X_3 and X_{3b} , both the results X_3^* and X_{3b}^* are slightly disconfirmed by the evidence, but the degree of disconfirmation of X_{3b}^* is lower. Belief analysis suggests considering the value 2.50×10^4 kg and, thus, supports decision making.

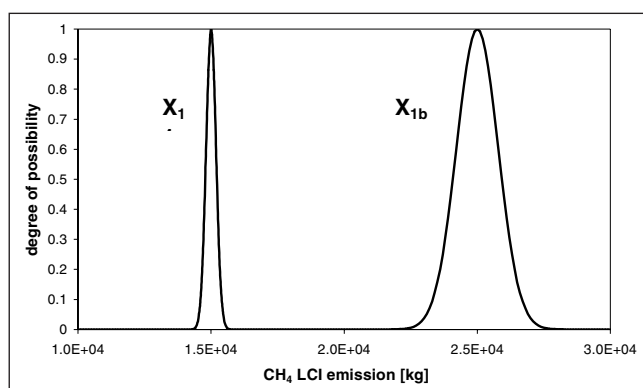


Fig. 1: Possibility distributions of X_1 and X_{1b}

3 Discussion and Outlook

The evaluation of belief, i.e. of the uncertainty on the true-ness of data (not on their values), can be of interest whenever two or more alternative options (e.g. LCI results, LCIA methods and parameters, ...) are possible, leading to different LCA results, and classical approaches of uncertainty

analysis or decision making do not provide enough information to choose. The need to invoke possibility theory arise from the axiomatic limitations of probabilities in managing belief, and that could mislead LCA calculations. By means of the evaluation of confidence intervals (and related indexes C), several LCA calculation chains leading to the same type of result (e.g. an ecotoxicity impact) in different ways could be compared in order to choose the chain whose degree of confirmation by the evidence is higher. Also, the LCA calculations not confirmed by evidence could be identified (e.g. by looking for abrupt changes in the value of C) and then modified so to improve the reliability of the final LCA results. This approach is time and resource consuming, as MonteCarlo simulation is, but could be worth applying in specific situations when stakes are high and it is difficult to make methodological choices.

The possibilistic approach for uncertainty analysis was initiated in a PhD project (Benetto 2002) and then improved and developed to widen its field of application to the risk assessment practice (e.g. Benetto et al. 2004). The authors are interested in the comments of the readers and look forward to seeing debate and discussion concerned with this paper.

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